**Rotation of Axes**

*X*

***x***

**

*P*(*x*, *y*)

*P*(*X*, *Y*)

****

****

***Y***

******

***X***

***x***

*r*

*Y*

**O**

Let, be a point under coordinate system. makes an angle with positive axis.

,

.

Suppose that the coordinate system is rotated through an angle θ keeping the origin fixed. If P has coordinates  referred to the new axes, then

And

Converting equation (1) and equation (2) into matrix form we get,

**General Equation of Second Degree**

The general equation of second degree is

 --------------- (1)

Where *a, h, b, g, f* and *c* are constants.

**Angle of rotation to remove *xy* term from general equation of second degree**

The general equation of second degree is

 --------------- (1)

Under rotation through an angle , the transformation equations are





and the general equation of second degree becomes





which can be written as



where













The transformed equation will be independent of the product term  if  i.e.





.

The nature of the **conics** represent by (1)



may be predicted by the sign of the discriminant

.

Results are summarized below:

|  |  |
| --- | --- |
| **Conditions on** | **Regular Conics** |
|  | Ellipse |
|  | Hyperbola |
|  | Parabola |

**To reduce to standard form, we may follow the following steps:**

Rotate the coordinate axes through an angle θ such that to get rid of *xy* term and then use translation to reduce to standard form.

**Example:** Find the angle of rotation to remove *xy* term fromand hence write the transformed equation. Also reduce them to standard form and sketch them showing both set of axes.

**Solution:** Identifying with  and and choosing θ such that



From the triangle we see that



The values of  and  can then be computed from the half-angle formulas:













The transformation equations become



and

Substitution in the original equation gives

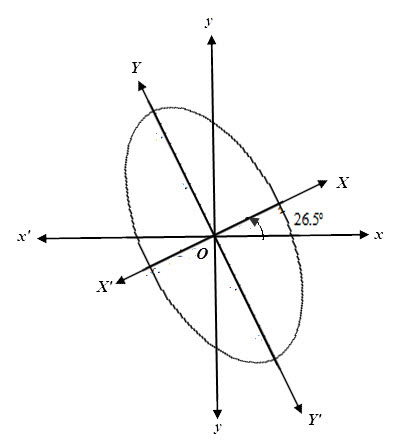


Collecting similar terms



or .

or.



**Example:** Find the angle of rotation to remove *xy* term fromand hence write the transformed equation.Also reduce them to standard form and sketch them showing both set of axes.

**Solution:** Identifying with  and and choosing θ such that



From the triangle we see that



The values of  and  can then be computed from the half-angle formulas:









7



The transformation equations become



and

Substitution in the original equation give

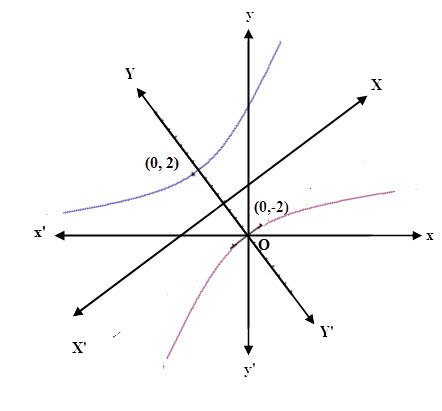


Collecting similar terms



or 

or  .



**Example:** Find the angle of rotation to remove *xy* term fromand hence write the transformed equation. Also reduce them to standard form and sketch them showing both set of axes.

**Solution:** Identifying with  and and choosing θ such that









1

From the triangle we see that



The transform equations are



and

Substitution in the original equation gives

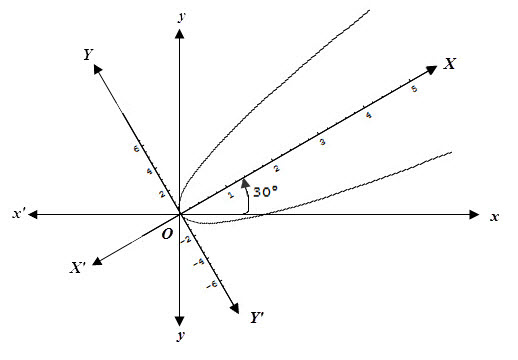


Collecting similar terms



or 

or  .

****

**Exercise**

1. Find the angle of rotation to remove *xy-*term from the following curves.

(a)  (b)  (c)

2. The Coordinate axes are rotated by the following given angle . Find the transformed equations of the following curves. Also reduce them to standard form and sketch them showing both set of axes.

(a) 7

(b) 5

(c)

(d)

(e)

(f)